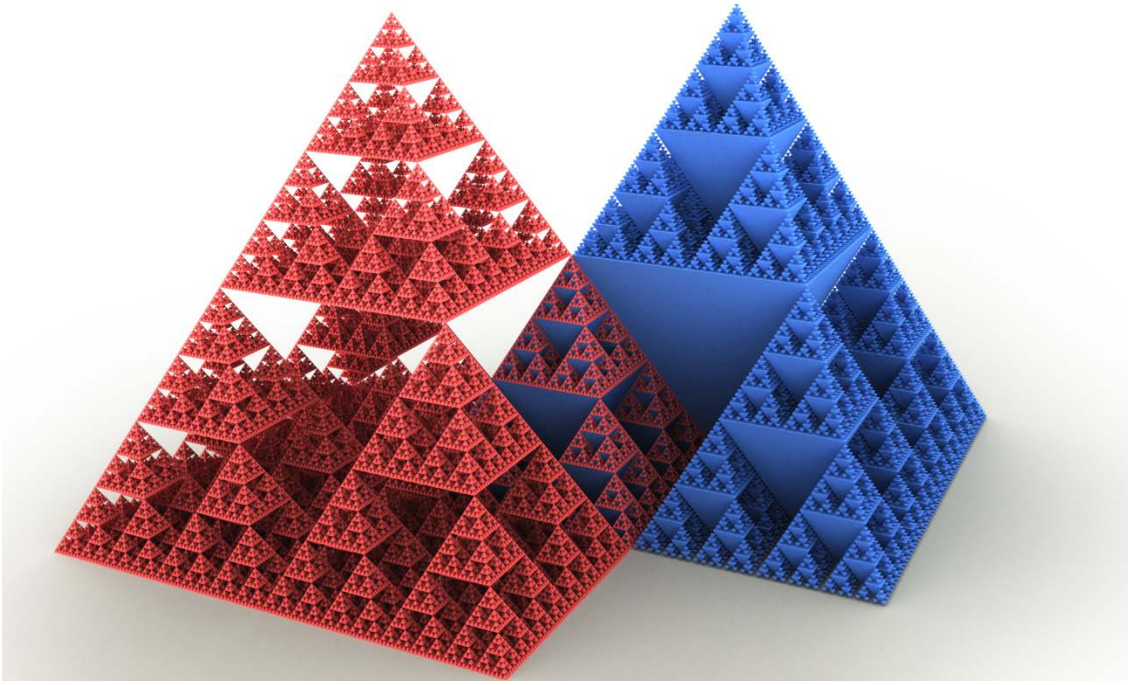
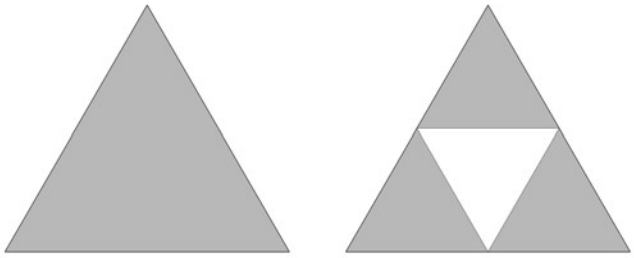


The Sierpinski Triangle

The Sierpinski triangle was rediscovered by Sierpinski in 1915. Sierpinski was held in such high regard by his contemporaries that one of the craters on the moon is named in his honor. Although Sierpinski was the first (known) to describe and analyze this object mathematically, similar figures were invented as early as the 13th century when it was used in the Cosmati mosaic tile work in the Cathedral of Agnani in Italy.

As shown below, the initiator is an equilateral triangle and the generator three equilateral triangles surrounding an empty space created by an equilateral triangle of the same size.



On the last page is an image of *isometric dot paper* which is particularly useful for creating equilateral triangles - and an image of the Initiator has been drawn for you.

1. Create Stage 2 of the Sierpinski triangle.

2. On a separate sheet, create Stage 3 of the Sierpinski triangle.

3. On a separate sheet, create Stage 4 of the Sierpinski triangle.

4. On a separate sheet, create Stage 5 of the Sierpinski triangle.

5. Consider the Initiator of the Sierpinski triangle to have unit area; i.e. $A = 1$. What is the area of each copy of the Initiator used to create the Generator?

6. What is the area of the Generator?

7. How many copies of the Generator are used to create Stage 2?

8. What is the area of each of the scaled copies of the Generator used in Stage 2?

9. What is the area of Stage 2?

10. How many copies of Stage 2 are used in creating Stage 3?

11. What is the area of each of the scaled copies of Stage 2 used in Stage 3?

12. What is the area of Stage 3?

13. Find an expression for the area of Stage n.

14. Make a table and/or plot data for values of n to analyze what happens to the area in Stage n for larger and larger values of n.

15. The Sierpinski triangle started from a 2-dimensional triangle. Based on your explorations, does it seem like the Sierpinski triangle is 2-dimensional? Explain.

What about the perimeter of the Sierpinski triangle? As results from Investigation **14** should show, the area of the Sierpinski triangle is zero. It is zero no matter what the units of area measurement is. Since you have drawn the stages of the Sierpinski triangle on dot paper, use the units of this paper to measure perimeter - each side of the Initiator is 32 units long.

16. What is the perimeter of the Initiator?

17. What is the perimeter of the Generator? (Note: Since the interior triangle has been removed, there is boundary on the inside which contributes to the perimeter as perimeter is a measure of the boundary.)

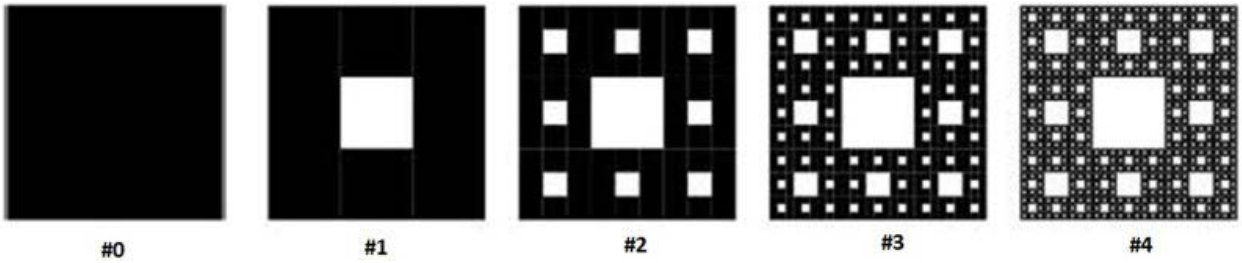
18. What is the perimeter of Stage 2?

19. What is the perimeter of Stage 3?

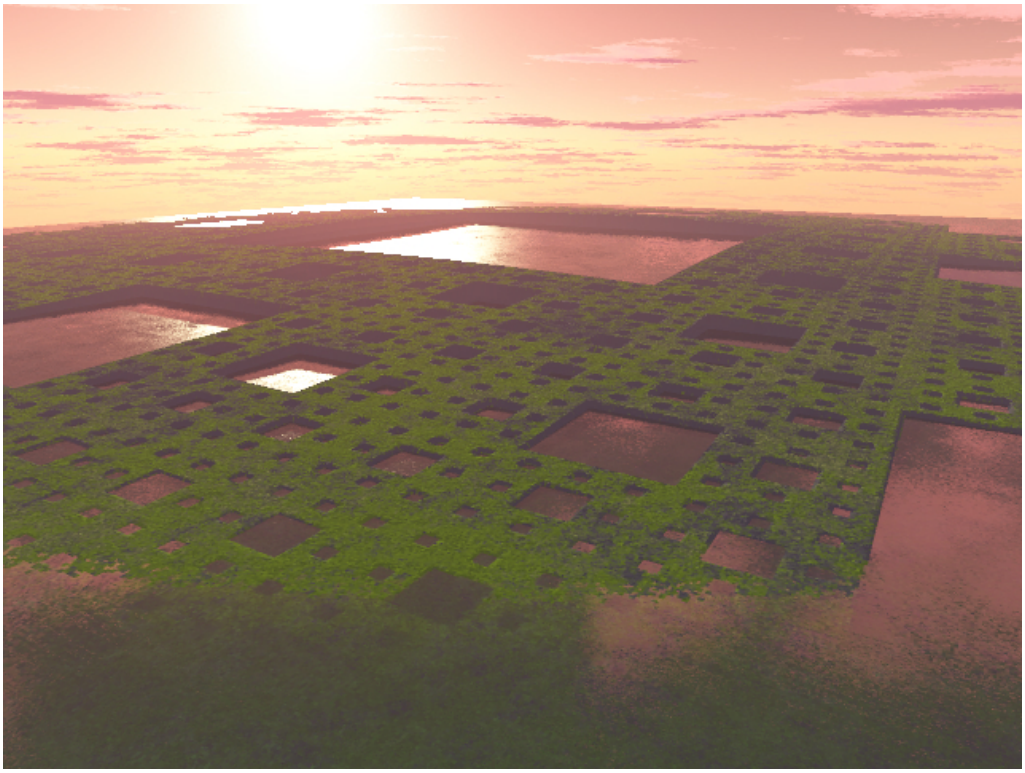
20. What is the perimeter of Stage 4?

21. What is the perimeter of Stage 5?

22. What can you conclude about the perimeter of the Sierpinski triangle? How does this result compare with the earlier analysis of the Menger Sponge?



23. Play around with the Sierpinski carpet. See how many iterations you can make. Refer to the initiator and generator above. You might want to use graph paper – and remember to start with a large stage 0.



Carpet Island by Paul Bourke